

Monopoly – Radical Markets

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1 Background

The game board⁰ – Rent increases¹ – Total money available² = 20,580 USD – Rules³

Key differentiators for COST-Monopoly⁴ = 200 USD received before the 10% taxation in the COST model, No Jail or Chance or Community Chest or Luxury/Income Tax locations – just properties + railroads + utilities – i.e. 28 units in total. All the probabilities then become simple, and the problems that remain are: Establishing the house rules, COST reserving, English auction pricing, and Resource management.

2 Review of Dice Probability

With 28 properties and an average roll value of 7, 4 opponent rolls will circumnavigate the board. Average roll is 7, so two unit blocks will on average be hit 2/7 for each player-circuit of the board.

```
python3
from collections import Counter
die = list(range(1,7))
rolls = Counter([ a+b for a in die for b in die ])
for a,b in rolls.items():
    print(a,b)
2 1
3 2          8 5
4 3          9 4
5 4          10 3
6 5          11 2
7 6          12 1
```

3 What are the House Rules?

3.1 Speeding?

Speeding just slows the game down.

3.2 What happens when I buy a place with a house on it?

If the house stays, then we have an upward building game. Otherwise, the gameplay will be nuclear strike oriented.

In the nuclear case, players should reserve higher, which should exacerbate cash flow problems.

4 Optimal Reserve Costs

All players will reserve at higher than the purchase cost – lest they lose money when others “steal” their properties.

4.1 Reserve more than other players can pay

Once you have a monopoly, reserve at: $\max(\text{other players cash})$

In the first round, the costliest situation is another player with all their initial cash and you have a Railroad monopoly: tax will be = $1500 \text{ reserve/Block} * -0.1 \text{ tax} + 200 \text{ tax subsidy} = 50$, so railroads can cost $1500 / 4 = 375$ per unit.

⁰ <http://bradfrost.com/blog/post/monopoly-photoshop-template/>

¹ <http://bellewether.me/monopoly-property-cards-template/>

² https://hasbro-new.custhelp.com/app/answers/detail/a_id/69/~how-much-money-comes-in-a-monopoly-game%3F

³ https://monopoly-game.net/Classic_Monopoly_Rules.html

⁴ <https://pbs.twimg.com/media/DwuRurMXQAEi1a3.jpg:large>

Also, provided that there is no collusion and all other players are intent on obtaining a monopoly of their own, then you can reserve at the max reserve of the other monopolies if that's less than max per-player cash.

NOTE: You must always increase your reserve before adding houses – otherwise others may buy properties out from under you.

4.2 Over-Reserving

Players will likely over-reserve because they don't want to lose their property. You want to be compensated enough for having a monopoly destroyed, any more is value thrown to the other players via the taxman – especially if the COST reallocation nukes any houses developed.

Players may reduce their reserve prices just prior to tax assessment, thereby creating a buying opportunity.

4.3 Using other players' reserve information

COST incentivizes players to reveal their property intentions via reserve pricing.

So, you want to reserve at the point where you are ambivalent between cash and holding the monopoly – i.e. where the monopoly benefit to you equals the benefit to you of the other player nuking their prospects should they buy you out.

We need a discount rate to compare the present values for cash and houses. The game is a race to max out houses first, and cash second.

First round, while cash abounds, players would pay a lot of interest for a marginal dollar more than OPs. Equivalently, you can earn a marginal dollar more by ensuring that OPs pay more for a dollar of earnings than you do.

5 Optimal Bidding for Properties

To yield an expected value per player-circuit of at least 50 USD per unit, the following are the max bids per unit per block:⁵

Block 1 is worth 135 per unit, Block 2 = 325, Block 3, 4 = 270, Block 5 = 175, Block 6 = 300, Block 7 = 275, and Block 8 = 500. These were calculated by subtracting the cost of building houses to yield an income of at least 50 and the cost of circumnavigating the board.

5.1 What is the point of ruin for each property?

When the property purchase prevents house building – average prices per property:

Block 1, 8 = 700, Block 2, 3, 4, 5, 6 = 450

5.2 Railroads and Utilities

Neither Railroads nor Utilities require development.

Railroads earn 25, 50, 100, 200 for the number owned. Railroads are on each side, therefore for the monopoly:
expected value per player-circuit = $(1 - (6/7) ** 4) * 200 = 92$

While for the 3-railroad duopoly:

$$EV/PC = (1 - (6/7) ** 3) * 100 = 37$$

And for the 2-railroad oligopoly:

$$EV/PC = (1 - (6/7) ** 2) * 50 = 13$$

Utilities – two total on opposite sides of the board – and when both are owned will rent at 70 USD average:

$$EV/PC = ((1 - (6/7) ** 2) * 70 = 18$$

Bid to win for the first N-1 units, then draw any bidders into over-paying for the last?

5.3 Frustrating other monopoly formation

If you have the cash flow, then bidding other units up to the point of ruin forces other players to fail, and should you win the 3rd unit when someone has controls the rest of the block, you can reserve it at a ruinous price.

⁵ In the first tax round (4 rolls per player), Block 1 is un-buyable because the first unit will never be rolled.

6 Resources

There are only 32 houses in the game, and 22 properties. The more players there are, the faster the house supply will exhaust.

7 Strategies

The third block may complete in the first round, its value changes given the number of players not yet past those units.

With N players left to roll, odds landing on 3rd monopoly (units 7,9,10) are at least 13/36 each player – players who roll 5 then 4 will also land – average rental income for the turn will be on average:

$$EV/PC = \sum_{n=7,8,9} odds(n) * income(n) = N * (150*6+150*4+180*3)/36$$

$$EV/PC = (150*6+150*4+180*3)/36 = 56$$

All property values floor at the mortgage value - 10 USD.

The number of players determines the cash crunch at the start:

$$8 * 1500 / 22 = \mathbf{545} \quad - \quad 7 * 1500 / 22 = \mathbf{477} \quad - \quad 6 * 1500 / 22 = \mathbf{409} \quad - \quad 5 * 1500 / 22 = \mathbf{340}$$

7.1 Subsequent Rounds

Always look for trades amidst others' in-completed blocks, and other forms of collusion.⁶

Player 1 sells last remaining colour needed for the group to player 2 for a dollar, then split the rental income.

Players may misallocate, which means that Blocks 1 and 8 might go quite cheaply.

7.2 Slumlord

Acquire Blocks 2 and 1 – rapidly build out houses to 4 each per unit, leaving 12 houses left for everyone else.

7.3 Scumlord

Buy the railroads for the immediate taxation on the other players. Just sit on your cash, serially buying then ransoming properties or stealing completed monopolies from those who under-reserve, and bidding heavily when someone will complete a Block. People will then over-pay and over-reserve, and you pick up the end-of-round transfer payments. When the weakest player bankrupts, you have the cash to bid when everyone else has committed their cash to properties.

7.4 Default

If the pricing at start is cheap, play Slumlord. If rich, then play Scumlord, then Rich Dad, Poor Dad.

If in range, acquire any Block 3 to 7, with max players these units generate the most cashflow.

7.5 What if there is more than one Informed Player?

Splitting the railroads causes EV/PC to drop to 37 (best case 1 other player) or worse, which neither want. Offer to buy the other IPs out, or a residual split, with a one round lease-buyback if cash crunch occurs. The negotiation needs to happen after all Railroads have been purchased, lest OPs bid ransom the unpurchased. Since we all will have overpaid, everyone is motivated to find a solution.

It will make it easier to get uninformed OPs to over-bid and over-reserve. However each extortion bidding is optimal with one IP – since the bids are sequential, then this should be a non-issue and easily observable.

⁶ <http://www.amnesta.net/other/monopoly/>

8 End States

8.1 Does the social dividend prevent total domination?

When the cash costs of circumnavigating the board are greater than the social dividend, that player is on the path to exiting the game.

When there is one monopoly and everyone else is on the dole, the monopoly reserves at max OP cash, paying 200 in tax but receives $200 + 200/N$ dole + 100 EV/PC while everyone else receives $200 + 200/N$ dole - 100 EV/PC. As N increases, the monopolist's position improves; where is the point when the monopolist position tips negative?

$$200 + 200/N + 100 * N = 200 + 200/N - 100$$

$$100N = -100$$

$$N = -1 \text{ - i.e. never.}$$

When there are two monopolies that reserve at max OP cash, the monopoly with less cash pays more into the social dividend than the one with more cash, which makes the situation worse for the less cash monopoly. Failure to reserve at max OP cash results in the other OP stealing the monopoly. Thus, the less cash monopoly can only win when its income is higher:

$$Ainc + 200 - Bcash0.1 + (Bcash+Acash)0.1/2 = Binc + 200 - Acash0.1 + (Bcash+Acash)0.1/2$$

$$Ainc - Bcash0.1 = Binc - Acash0.1$$

$$Acash0.1 - Bcash0.1 = Binc - Ainc$$

$$Acash - Bcash = 10 (Binc - Ainc)$$

The less cash monopoly wins when the cash spread is less than 10 times the income spread.